## Hoofdstuk 2: Kansrekening

### 2.1 Kansen

**Opgave 1:**
De kans dat ze 7 gooit is groter, want ze kan op zes manieren 7 gooien: 1-6, 2-5, 3-4, 4-3, 5-2, 6-1.
Ze kan op 4 manieren 9 gooien: 3-6, 4-5, 5-4, 6-3.

**Opgave 2:**

a. $\frac{3}{36}$

b. $\frac{18}{36}$

c. $\frac{6}{36}$

d. $\frac{15}{36}$

e. $\frac{16}{36}$

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 1 | 2 | 3 | 4 | 5 | 6 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | | | | | | |
| 5 | | | | | | x |
| 4 | | | | | x | x |
| 3 | | | | x | x | x |
| 2 | | | x | x | x | x |
| 1 | | x | x | x | x | x |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave d

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | | | | | x | x |
| 5 | | | | x | x | x |
| 4 | | | x | x | x | |
| 3 | | x | x | x | | |
| 2 | x | x | x | | | |
| 1 | x | x | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave e

**Opgave 3:**

a. $\frac{8}{36}$

b. $\frac{4}{36}$

c. $\frac{33}{36}$

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | | | | x | | |
| 5 | | | x | | | |
| 4 | | x | | | | x |
| 3 | x | | | | x | |
| 2 | | | | x | | |
| 1 | | | x | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave a

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | | x | | | | |
| 5 | | | | | | |
| 4 | | | x | | | |
| 3 | | | | x | | |
| 2 | | | | | | x |
| 1 | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave b

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | x | x | x | x | | |
| 5 | x | x | x | x | x | |
| 4 | x | x | x | x | x | x |
| 3 | x | x | x | x | x | x |
| 2 | x | x | x | x | x | x |
| 1 | x | x | x | x | x | x |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave c

**Opgave 4:**

a. $\frac{4}{32}$

b. $\frac{4}{32}$

c. $\frac{9}{32}$

d. $\frac{7}{32}$

| | | | | |
|---|---|---|---|---|
| 8 | | | | |
| 7 | | | | |
| 6 | | | | |
| 5 | | | | |
| 4 | | | | x |
| 3 | | | x | |
| 2 | | x | | |
| 1 | x | | | |
| | 1 | 2 | 3 | 4 |

opgave a

| | | | | |
|---|---|---|---|---|
| 8 | | | | |
| 7 | x | | | |
| 6 | | x | | |
| 5 | | | x | |
| 4 | | | | x |
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |
| | 1 | 2 | 3 | 4 |

opgave b

| | | | | |
|---|---|---|---|---|
| 8 | | x | x | x |
| 7 | | | x | x |
| 6 | | | x | x |
| 5 | | | | x |
| 4 | | | | x |
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |
| | 1 | 2 | 3 | 4 |

opgave c

| | | | | |
|---|---|---|---|---|
| 8 | | | | |
| 7 | | | | |
| 6 | | | | |
| 5 | | | | x |
| 4 | | | x | |
| 3 | | x | | x |
| 2 | x | | x | |
| 1 | | x | | |
| | 1 | 2 | 3 | 4 |

opgave d

**Opgave 5:**

a. niet iedere sector is even groot

b. $\frac{1}{6}$

**Opgave 6:**

totaal: $2^4 = 16$ mogelijkheden

a. 4 keer kop of 4 keer munt dus: $\binom{4}{0} + \binom{4}{4} = 1 + 1 = 2$

$P(4\ keer\ hetzelfde) = \frac{2}{16}$

b. 1 keer kop: $\binom{4}{1} = 4$

$P(1\ keer\ kop) = \frac{4}{16}$

c. 2 keer munt of 3 keer munt of 4 keer munt, dus: $\binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 6 + 4 + 1 = 11$

$P(meer\ dan\ 1\ keer\ munt) = \frac{11}{16}$

**Opgave 7:**

totaal: $6^3 = 216$ mogelijkheden

a. 1-1-3 kan op 3 manieren
1-2-2 kan op 3 manieren
dus $\frac{6}{216}$

b. som = 3 :1-1-1 kan op 1 manier
som = 4 :1-1-2 kan op 3 manieren
som = 5 : kan op 6 manieren (zie opgave a)
som = 6 :1-1-4 kan op 3 manieren
1-2-3 kan op 6 manieren
2-2-2 kan op 1 manier
dus $\frac{20}{216}$

c. 3 keer 1, of 3 keer 2, of 3 keer 3, of 3 keer 4, of 3 keer 5, of 3 keer 6
dus $\frac{6}{216}$

**Opgave 8:**

totaal: $6^4 = 1296$ mogelijkheden

a. 6-6-6-4 kan op 4 manieren
6-6-5-5 kan op $\frac{4!}{2! \cdot 2!} = 6$ manieren
dus $\frac{10}{1296}$

b. som = 4: 1-1-1-1 kan op 1 manier
som = 5: 1-1-1-2 kan op 4 manieren
som = 6: 1-1-1-3 kan op 4 manieren
1-1-2-2 kan op 6 manieren
dus $\frac{15}{1296}$

c. 1-1-1-4 kan op 4 manieren
1-1-2-2 kan op 6 manieren
dus $\frac{10}{1296}$

**Opgave 9:**

totaal: $2^6 = 64$ mogelijkheden

a. $\binom{6}{5} = 6$ dus $\frac{6}{64}$

b. $\binom{6}{3} = 20$ dus $\frac{20}{64}$

c. 0 keer kop, of 1 keer kop, of 2 keer kop: $\binom{6}{0} + \binom{6}{1} + \binom{6}{2} = 1 + 6 + 15 = 22$

dus $\frac{22}{64}$

## 2.2 Empirische kansen

**Opgave 10:**

a.

| aantal worpen $N$ | 30 | 60 | 120 | 180 | 240 | 300 |
|---|---|---|---|---|---|---|
| frequentie $f$ | 7 | 9 | 22 | 34 | 41 | 48 |
| relatieve frequentie $\frac{f}{N}$ | 0,23 | 0,15 | 0,18 | 0,19 | 0,17 | 0,16 |

b. 100 keer

c. $\frac{1}{6}$

d. hij verwacht 3 keer zes ogen te gooien, omdat hij niet zo vaak gooit, kan het dus wel.

e. nee, je verwacht dan 300 keer zes ogen te ogen, dus de afwijking is heel erg groot.

**Opgave 11:**

a.

| aantal keer gooien $N$ | 50 | 100 | 150 | 200 | 250 | 300 | 500 | 1000 |
|---|---|---|---|---|---|---|---|---|
| frequentie $f$ | 31 | 61 | 89 | 114 | 141 | 174 | 282 | 579 |
| relatieve frequentie $\frac{f}{N}$ | 0,62 | 0,61 | 0,59 | 0,57 | 0,56 | 0,58 | 0,56 | 0,58 |

b. 0,58

c. er zijn wel twee mogelijkheden, maar deze blijken in de praktijk niet even vaak voor te komen, dus de kans dat de punaise met de punt omhoog ligt is niet gelijk aan 0,5.

**Opgave 12:**

a. $2\,328\,757 + 1\,986\,351 + 186\,294 = 4\,501\,402$

totaal: $6\,991\,991$

dus $\frac{4\,501\,402}{6\,991\,991} = 0{,}644$

b. $\frac{1\,544\,585 + 2\,328\,757}{6\,991\,991} = 0{,}544$

**Opgave 13:**

a. $\frac{19+7}{57} = 0{,}46$

b. $\frac{30+9+5}{57} = 0{,}77$

c. $\frac{13+21+12}{57} = 0{,}81$

**Opgave 14:**

a. $0{,}2 + 0{,}2 = 0{,}4$

b. $0{,}15 + 0{,}25 = 0{,}4$

c. $0{,}15 + 0{,}05 = 0{,}2$

**Opgave 15:**

a. empirisch
b. theoretisch
c. empirisch
d. theoretisch
e. empirisch

## 2.3 Voorwaardelijke kansen

**Opgave 23:**

a. $\frac{58}{64} = 0{,}906$

b. $\frac{47}{51} = 0{,}922$

c. $\frac{105}{115} = 0{,}913$

**Opgave 24:**

a. $\frac{9+12+4}{285} = \frac{25}{285} = 0{,}088$

b. $\frac{10+16}{43} = \frac{26}{43} = 0{,}605$

c. $\frac{4+5+2+7}{27} = \frac{18}{27} = 0{,}667$

d. $\frac{5+2+7+3+5+2+1+5}{50+37+43} = \frac{30}{130} = 0{,}231$

**Opgave 25:**

a. $\frac{2581}{8527} = 0{,}303$

b. $\frac{2970}{8527} = 0{,}348$

c. $\frac{982}{2088} = 0{,}470$

d. N-Z of O-W of Z-N of W-O

$\frac{53+1711+51+1682}{8572} = \frac{3497}{8527} = 0{,}410$

e. N-O of O-Z of Z-W of W-N

$\frac{1053+154+830+408}{8527} = \frac{2445}{8527} = 0{,}287$

f. $\frac{408}{2581} = 0{,}158$

g. $\frac{2581}{8527} \cdot 7520 = 2276$

**Opgave 26:**

a. $\frac{855134}{983378} = 0{,}859$

b. $983378 - 967252 = 16126$ dus $\frac{16126}{983378} = 0{,}016$

c. $\frac{11472}{344752} = 0{,}033$

d. $1000000 - 983378 = 16622$ dus $\frac{16622}{1000000} = 0{,}017$

e. $659273 - 344752 = 314521$ dus $\frac{314521}{845134} = 0{,}372$

**Opgave 27:**

a. $\frac{82}{496} = 0{,}165$

b. $\frac{82-59}{496} = 0{,}046$

c. $\frac{59}{169} = 0{,}349$

d. $\frac{59}{118} = 0{,}500$

e. $\frac{42-32}{42} = \frac{10}{42} = 0{,}238$

f. $\frac{118-42}{118} = \frac{76}{118} = 0{,}644$

**Opgave 28:**

a. bereken de kans date en treinpassagier jonger dan 20 jaar is.
b. bereken de kans dat een treinpassagier 40 jaar of ouder is.
c. bereken de kans dat een treinpassagier uit de leeftijdscategorie $20- < 40$ een voordeelurenkaart heeft.
d. bereken de kans dat een treinpassagier zonder voordeelurenkaart uit de leeftijdscategorie $20- < 40$ komt.
e. bereken de kans dat een treinpassagier een voordeelurenkaart heeft en jonger is dan 20 jaar.
f. bereken de kans dat een treinpassagier uit de leeftijdscategorie $20- < 60$ een voordeelurenkaart heeft.

**Opgave 29:**

a.

| | A | B | C | |
|---|---|---|---|---|
| wel antwoord | 570 | 690 | 147 | 1407 |
| niet antwoord | 30 | 60 | 3 | 93 |
| | 600 | 750 | 150 | 1500 |

$\frac{1407}{1500} = 0{,}938$

b. $\frac{570}{1407} = 0{,}405$

c. $\frac{3}{93} = 0{,}032$

**Opgave 30:**

| | A | B | C | |
|---|---|---|---|---|
| wel op post | 0,357 | 0,20 | 0,2706 | 0,8276 |
| niet op post | 0,063 | 0,05 | 0,0594 | 0,1724 |
| | 0,42 | 0,25 | 0,33 | 1 |

a. 0,1724

b. $\frac{0{,}063}{0{,}1724} = 0{,}365$

c. 0,357

**Opgave 31:**

| | wel tbc | niet tbc | |
|---|---|---|---|
| positief | 1,96 | 999,98 | 1001,94 |
| negatief | 0,04 | 8998,02 | 8998,06 |
| | 2 | 99998 | 10000 |

a. $\frac{1{,}96}{1001{,}94} = 0{,}0020$

b. $0{,}0020 \cdot 50 = 0{,}1$

**Opgave 32:**

$P(Rh^+) = \frac{1700}{2000} = 0{,}85$

$P(Rh^+ \text{ onder de voorwaarde bloedgroep O}) = \frac{1250}{1500} = 0{,}833$

Deze kansen zijn niet gelijk dus afhankelijk.

**Opgave 33:**

a.

| | A | geen A | |
|---|---|---|---|
| Rh + | 51 | 119 | 170 |
| Rh ─ | 9 | 21 | 30 |
| | 60 | 140 | 200 |

b. $\frac{9}{200} = 0{,}045$

c. $\frac{51}{170} = 0{,}300$

## 2.4 Het vaasmodel

**Opgave 34:**

a. als je de eerste persoon gekozen hebt, zijn er nog maar 9 mogelijkheden voor de tweede persoon en nog maar 8 mogelijkheden voor de derde persoon.

b. de volgorde waarin je de personen kiest, is niet van belang.

c. $\binom{10}{3} = 120$

**Opgave 35:**

$$\frac{\binom{7}{5} \cdot \binom{8}{0}}{\binom{15}{5}}$$

**Opgave 36:**

a. $\frac{\binom{7}{3}}{\binom{21}{3}} = 0{,}026$

b. $\frac{\binom{15}{3}}{\binom{21}{3}} = 0{,}342$

c. $\frac{\binom{7}{2} \cdot \binom{8}{1}}{\binom{21}{3}} = 0{,}126$

d. $\frac{\binom{8}{2} \cdot \binom{13}{1}}{\binom{21}{3}} = 0{,}274$

**Opgave 37:**

a. $\frac{\binom{32}{3} \cdot \binom{12}{3}}{\binom{62}{6}} = 0{,}0178$

b. $\frac{\binom{30}{6}}{\binom{62}{6}} = 0{,}0097$

c. $\frac{\binom{32}{4} \cdot \binom{30}{2}}{\binom{62}{6}} = 0{,}2545$

d. $\frac{\binom{18}{1} \cdot \binom{44}{5}}{\binom{62}{6}} = 0{,}3180$

**Opgave 38:**

a. $P(\text{0 blauw}) = \frac{\binom{10}{3}}{\binom{16}{3}} = 0{,}214$

$$P(1 \text{ blauw}) = \frac{\binom{6}{1}\cdot\binom{10}{2}}{\binom{16}{3}} = 0{,}482$$

$$P(2 \text{ blauw}) = \frac{\binom{6}{2}\cdot\binom{10}{1}}{\binom{16}{3}} = 0{,}268$$

$$P(3 \text{ blauw}) = \frac{\binom{6}{3}}{\binom{16}{3}} = 0{,}036$$

| aantal blauw | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| kans | 0,214 | 0,482 | 0,268 | 0,036 |

b. 1 ; alle kansen samen zijn altijd 1.

**Opgave 39:**

a. 1 rood, 5 blauw, 54 wit
hij pakt 5 knikkers

b. $\frac{\binom{5}{2}\cdot\binom{54}{3}}{\binom{60}{5}} = 0{,}045$

c. $\frac{\binom{1}{1}\cdot\binom{5}{1}\cdot\binom{54}{3}}{\binom{60}{5}} = 0{,}023$

**Opgave 40:**

a. $\frac{\binom{10}{1}\cdot\binom{30}{2}}{\binom{40}{3}} = 0{,}440$

b. $\frac{\binom{7}{2}\cdot\binom{30}{2}}{\binom{40}{4}} = 0{,}100$

c. $\frac{\binom{30}{7}}{\binom{40}{7}} = 0{,}109$

d. $\frac{\binom{10}{4}}{\binom{40}{4}} = 0{,}002$

**Opgave 41:**

a. $\frac{\binom{9}{6}}{\binom{15}{6}} = 0{,}017$

b. $\frac{\binom{6}{3}\cdot\binom{9}{3}}{\binom{15}{6}} = 0{,}336$

**Opgave 42:**

a. $\frac{\binom{1}{1}\cdot\binom{1}{1}\cdot\binom{1}{1}\cdot\binom{1}{1}}{\binom{26}{4}} = 6{,}7\cdot10^{-5}$

b. $\frac{\binom{1}{1}\cdot\binom{1}{1}\cdot\binom{1}{1}\cdot\binom{23}{1}}{\binom{26}{4}} = 0{,}00154$

c. $\frac{\binom{1}{1}\cdot\binom{1}{1}\cdot\binom{24}{2}}{\binom{26}{4}} = 0{,}0185$

**Opgave 43:**

a. $\frac{\binom{20}{7}}{\binom{41}{7}} = 0{,}0034$

b. $\frac{\binom{19}{7}}{\binom{41}{7}} = 0{,}0022$

c. $\frac{\binom{36}{7}}{\binom{41}{7}} = 0{,}3713$

d. $\frac{\binom{36}{6}\cdot\binom{1}{1}}{\binom{41}{7}} = 0{,}0866$

e. $\frac{\binom{1}{1}\cdot\binom{1}{1}\cdot\binom{24}{5}}{\binom{41}{7}} = 0{,}0019$

**Opgave 44:**

$\frac{\binom{18}{4}}{\binom{20}{4}} = 0{,}6316$

**Opgave 45:**

$\frac{\binom{18}{18}}{\binom{20}{18}} = 0{,}0053$

**Opgave 46:**

$\frac{\binom{490}{25}}{\binom{500}{25}} = 0{,}5959$

## 2.5 De somregel en de complementregel.

**Opgave 47:**

a. $P(\text{som}=3)=\frac{2}{36}$ (1-2 of 2-1)

$P(\text{som}=4)=\frac{3}{36}$ (1-3 of 2-2 of 3-1)

$P(\text{som}=3 \text{ of som}=4)=\frac{5}{36}$ (1-2 of 2-1 of 1-3 of 2-2 of 3-1)

b. ja

c. $P(\text{som}=4)=\frac{3}{36}$ (1-3 of 2-2 of 3-1)

$P(\text{product}=4)=\frac{3}{36}$ (1-4 of 2-2 of 4-1)

$P(\text{som}=4 \text{ of product}=4)=\frac{5}{36}$ (1-3 of 2-2 of 3-1 of 1-4 of 4-1)

Dus niet gelijk

**Opgave 48:**

De noemers zijn gelijknamig dus mag je eerst de tellers optellen en daarna pas delen door de noemer. Voordeel: minder werk.

**Opgave 49:**

a. $\frac{\binom{4}{2}\cdot\binom{6}{1}+\binom{4}{3}}{\binom{10}{3}}=0{,}333$

b. $P(\text{minder dan 2 groene})=P(0 \text{ of 1 groene})=\frac{\binom{6}{3}+\binom{6}{2}\cdot\binom{4}{1}}{\binom{10}{3}}=0{,}667$

**Opgave 50:**

a. $P(0 \text{ of 1 meisje})=\frac{\binom{13}{4}+\binom{13}{3}\cdot\binom{15}{1}}{\binom{28}{4}}=0{,}244$

b. $\frac{\binom{13}{1}\cdot\binom{15}{3}+\binom{13}{2}\cdot\binom{15}{2}\cdot\binom{13}{3}\cdot\binom{15}{1}}{\binom{28}{4}}=0{,}898$

**Opgave 51:**

a. $\frac{\binom{29}{9}\cdot\binom{24}{1}+\binom{29}{10}}{\binom{53}{10}}=0{,}0134$

b. $\frac{\binom{37}{10}+\binom{37}{9}\cdot\binom{16}{1}+\binom{37}{8}\cdot\binom{16}{2}}{\binom{53}{10}}=0{,}3575$

c. $\frac{\binom{5}{2}\cdot\binom{48}{8}}{\binom{53}{10}}=0{,}1935$

**Opgave 52:**

a. $\frac{\binom{10}{4}}{\binom{22}{4}}=0{,}0287$

b. $\frac{\binom{10}{3}\cdot\binom{12}{1}+\binom{10}{2}\cdot\binom{12}{2}+\binom{10}{1}\cdot\binom{12}{3}+\binom{12}{4}}{\binom{22}{4}} = 0{,}9713$

**Opgave 53:**

a. $P(\text{minstens 1 prijs}) = 1 - P(\text{geen prijs}) = 1 - \frac{\binom{21}{3}}{\binom{25}{3}} = 0{,}4217$

b. $P(\text{niet 3 prijzen}) = 1 - P(\text{3 prijzen}) = 1 - \frac{\binom{4}{3}}{\binom{25}{3}} = 0{,}9983$

c. $\frac{\binom{4}{2}\cdot\binom{21}{1}}{\binom{25}{3}} = 0{,}0548$

d. $\frac{\binom{21}{3}}{\binom{25}{3}} = 0{,}5783$

**Opgave 54:**

a. $P(\text{som} \neq 5) = 1 - P(\text{ som} = 5) = 1 - \frac{6}{216} = \frac{210}{216} = 0{,}972$

b. $P(\text{som} < 17) = 1 - P(\text{ som} \geq 17) = 1 - \frac{4}{216} = \frac{212}{216} = 0{,}981$

**Opgave 55:**

a. $P(\text{minstens 1 groene}) = 1 - P(\text{geen groen}) = 1 - \frac{\binom{9}{3}}{\binom{12}{3}} = 0{,}618$

b. $P(\text{hoogstens 2 blauw}) = 1 - P(\text{3 blauw}) = 1 - \frac{\binom{5}{3}}{\binom{12}{3}} = 0{,}955$

c. $\frac{\binom{4}{1}\cdot\binom{3}{1}\cdot\binom{5}{1}}{\binom{12}{3}} = 0{,}273$

d. $\frac{\binom{4}{3}+\binom{3}{3}+\binom{5}{3}}{\binom{12}{3}} = 0{,}068$

**Opgave 56:**

a. het complement van geen groene is minstens 1 groene.
b. het complement van gelijke kleuren is niet allemaal gelijke kleuren.
c. het complement van meer dan 2 rood is hoogstens 2 rood.
d. het complement van hoogstens 3 wit is minstens 4 wit.

**Opgave 57:**

a. $P(\text{minstens 1 barst}) = 1 - P(\text{geen barst}) = 1 - \frac{\binom{46}{10}}{\binom{50}{10}} = 0{,}6032$

b. $\frac{\binom{4}{4}\cdot\binom{46}{6}}{\binom{50}{10}} = 9{,}1\cdot 10^{-4}$

**Opgave 58:**

a. $P(\text{minstens 2 bestuursleden}) = 1 - P(\text{hoogstens 1 bestuurslid})$

$$= 1 - \left( \frac{\binom{59}{5} + \binom{59}{4}\cdot\binom{6}{1}}{\binom{65}{5}} \right) = 0{,}0633$$

b. $P(\text{minstens 1 keer supermarkt}) = 1 - P(\text{geen supermarkt})$

$$= 1 - \frac{\binom{57}{5}}{\binom{65}{5}} = 0{,}4931$$

c. er zijn 8 leden van de supermarkt en nog 4 bestuursleden die geen lid zijn van de supermarkt.

$\frac{\binom{53}{3}}{\binom{65}{5}} = 0{,}3474$

**Opgave 59:**

a. $\frac{\binom{20}{6}\cdot\binom{10}{2} + \binom{20}{7}\cdot\binom{10}{1} + \binom{20}{8}}{\binom{30}{8}} = 0{,}452$

b. $P(\text{minder dan 7 jongens}) = 1 - P(\text{minstens 7 jongens}) = 1 - \frac{\binom{12}{7}\cdot\binom{18}{1} + \binom{12}{8}}{\binom{30}{8}} = 0{,}0025$

c. $\frac{\binom{13}{3}\cdot\binom{17}{5}}{\binom{30}{4}} = 0{,}3024$

**Opgave 60:**

a. $\frac{\binom{24}{5}}{\binom{30}{5}} = 0{,}2983$

b. $P(\text{minstens 2 defect}) = 1 - P(\text{hoogstens 1 defect}) = 1 - \left( \frac{\binom{24}{5} + \binom{24}{4}\cdot\binom{6}{1}}{\binom{30}{5}} \right) = 0{,}7457$

c. $\frac{\binom{24}{3}\cdot\binom{6}{2} + \binom{24}{4}\cdot\binom{6}{1} + \binom{24}{5}}{\binom{30}{5}} = 0{,}9587$

## 2.6 De productregel.

**Opgave 61:**

a. 3 van de 4 kinkkers zijn rood

b. $\frac{2}{3}$

c. $4 \cdot 3 = 12$ ; 6 keer rr

d. $\frac{6}{12}$

e. $\frac{3}{4} \cdot \frac{2}{3} = \frac{6}{12}$

**Opgave 62:**

a. $\frac{5}{10} \cdot \frac{2}{5} = \frac{10}{50}$

b. $\frac{2}{10} \cdot \frac{2}{5} = \frac{4}{50}$

c. $\frac{5}{10} \cdot \frac{1}{5} = \frac{5}{50}$

d. $\frac{7}{10} \cdot \frac{3}{5} = \frac{21}{50}$

e. $\frac{8}{10} \cdot 1 = \frac{8}{10}$

**Opgave 63:**

a. $\frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{2}{24}$

b. $\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{6}{24}$

c. cbb of bcb dus $\frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} + \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{3}{24}$

d. 0

**Opgave 64:**

a. $\left(\frac{4}{6}\right)^3 = \frac{64}{216}$

b. $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$

c. $\left(\frac{2}{6}\right)^3 = \frac{8}{216}$

**Opgave 65:**

a. $\left(\frac{3}{4}\right)^4 = 0{,}3164$

b. $\left(\frac{2}{4}\right)^4 = 0{,}0625$

c. $P(\text{4 keer 1}) = \left(\frac{1}{4}\right)^4 = 0{,}0039$

**Opgave 66:**

a. empirisch

b. $0{,}6 \cdot 0{,}5 \cdot 0{,}8 = 0{,}24$

c. $0{,}4 \cdot 0{,}2 \cdot 0{,}2 = 0{,}016$

d. $P(\text{soep,vis,ijs}) = 0{,}6 \cdot 0{,}3 \cdot 0{,}8 = 0{,}144$ dus $0{,}144 \cdot 500 = 72$

**Opgave 67:**

a. $0{,}4^2 = 0{,}16$

b. $0{,}7 \cdot 0{,}2 = 0{,}14$

c. $0{,}7 \cdot 0{,}7 = 0{,}49$

d. $0{,}35 \cdot 0{,}12 = 0{,}042$

**Opgave 68:**

a. $\frac{2}{4} \cdot \frac{1}{3} = \frac{2}{12}$

b. $\frac{3}{4} \cdot \frac{2}{3} = \frac{6}{12}$

c. $\frac{2}{4} \cdot \frac{2}{3} + \frac{2}{4} \cdot \frac{1}{3} = \frac{6}{12}$

d. 2-2 of 3-1 dus $\frac{1}{4} \cdot \frac{1}{3} + \frac{2}{4} \cdot \frac{1}{3} = \frac{3}{12}$

e. $P(\text{minstens 1 keer 3}) = 1 - P(\text{geen 3}) = 1 - \frac{2}{4} \cdot \frac{2}{3} = \frac{8}{12}$

**Opgave 69:**

a. wwb of wbw of bww dus $\frac{2}{5} \cdot \frac{2}{6} \cdot \frac{3}{4} + \frac{2}{5} \cdot \frac{4}{6} \cdot \frac{1}{4} + \frac{3}{5} \cdot \frac{2}{6} \cdot \frac{1}{4} = \frac{26}{120} = 0{,}217$

b. $\frac{2}{5} \cdot \frac{2}{6} \cdot \frac{1}{4} = \frac{4}{120} = 0{,}033$

c. $P(\text{minstens 1 wit}) = 1 - P(\text{geen wit}) = 1 - \frac{3}{5} \cdot \frac{4}{6} \cdot \frac{3}{4} = \frac{84}{120} = 0{,}7$

d. bbb of wbb of bwb of bbw
$\frac{3}{5} \cdot \frac{4}{6} \cdot \frac{3}{4} + \frac{2}{5} \cdot \frac{4}{6} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{2}{6} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{4}{6} \cdot \frac{1}{4} = \frac{90}{120} = 0{,}75$

**Opgave 70:**

a. $\frac{3}{8} \cdot \frac{1}{8} \cdot \frac{2}{8} = \frac{6}{512} = 0{,}0117$

b. $\frac{2}{8} \cdot \frac{6}{8} \cdot \frac{2}{8} + \frac{2}{8} \cdot \frac{1}{8} \cdot \frac{3}{8} + \frac{2}{8} \cdot \frac{6}{8} \cdot \frac{3}{8} = \frac{66}{512} = 0{,}1289$

c. $\frac{7}{8} \cdot \frac{8}{8} \cdot \frac{7}{8} = \frac{392}{512} = 0{,}7656$

d. $\frac{1}{8} \cdot \frac{8}{8} \cdot \frac{7}{8} + \frac{7}{8} \cdot \frac{8}{8} \cdot \frac{1}{8} = \frac{112}{512} = 0{,}2188$

e. $P(\text{minstens 1 peer}) = 1 - P(\text{geen peer}) = 1 - \frac{5}{8} \cdot \frac{7}{8} \cdot \frac{6}{8} = 1 - \frac{210}{512} = \frac{302}{512} = 0{,}5898$

**Opgave 71:**

a. $0{,}4 \cdot 0{,}25 = 0{,}1$

b. $0{,}42 \cdot 0{,}6 \cdot 0{,}4 \cdot 0{,}75 = 0{,}0756$

c. $P(\text{wordt niet 3 jaar}) = 1 - P(\text{wordt wel 3 jaar}) = 1 - 0{,}42 \cdot 0{,}6 \cdot 0{,}4 = 0{,}8992$

## 2.7 Diagnostische toets

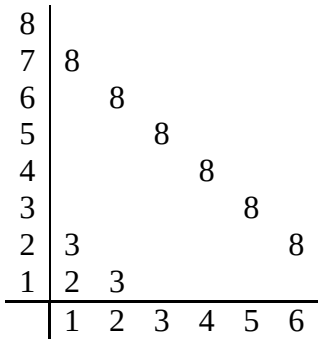
**Opgave 1:**

a. $\frac{6}{48}$

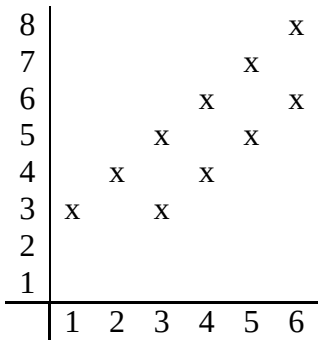
b. $\frac{3}{48}$

c. $\frac{10}{48}$

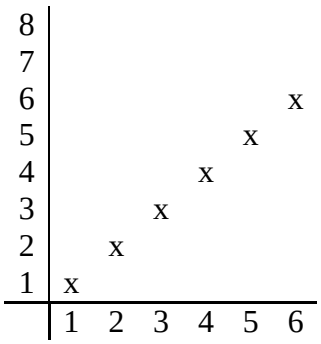
d. $\frac{6}{48}$




opgave a,b




opgave c




opgave d

**Opgave 2:**

Totaal $4^4 = 256$ mogelijkheden

a. 1-1-4-4: $\frac{4!}{2! \cdot 2!} = 6$ manieren

1-2-2-4: $\frac{4!}{2!} = 12$ manieren

2-2-2-2: 1 manier

dus $\frac{19}{256}$

b. som = 14: 4-4-4-2: 4 manieren

4-4-3-3: $\frac{4!}{2! \cdot 2!} = 6$ manieren

som = 15: 4-4-4-3: 4 manieren

som = 16: 4-4-4-4: 1 manier

dus $\frac{15}{256}$

**Opgave 3:**

a. $\frac{159 + 33}{215} = 0{,}983$

b. $\frac{12}{215} = 0{,}056$

c. $\frac{15 + 5}{215} = 0{,}093$

**Opgave 4:**

a. $\frac{15 + 59}{80} = 0{,}925$

b. $\frac{6}{33} = 0{,}182$

c. $\frac{59 + 49 + 6 + 12}{80 + 66} = \frac{126}{146} = 0{,}863$

d. $\dfrac{49+51+12+15}{215}=0{,}591$

**Opgave 5:**

$0{,}15\cdot 0{,}03=0{,}0045$

**Opgave 6:**

a. $\dfrac{\binom{5}{2}\cdot\binom{7}{2}\cdot\binom{9}{2}}{\binom{21}{6}}=0{,}139$

b. $\dfrac{\binom{12}{6}}{\binom{21}{6}}=0{,}017$

c. $\dfrac{\binom{5}{2}\cdot\binom{16}{4}}{\binom{21}{6}}=0{,}335$

**Opgave 7:**

a. $\dfrac{\binom{33}{4}}{\binom{40}{4}}=0{,}4478$

b. $\dfrac{\binom{7}{2}\cdot\binom{33}{2}}{\binom{40}{4}}=0{,}1213$

c. $\dfrac{\binom{1}{1}\cdot\binom{6}{1}\cdot\binom{33}{2}}{\binom{40}{4}}=0{,}0347$

**Opgave 8:**

a. $P(\text{minstens 1 rood})=1-P(\text{geen rood})=1-\dfrac{\binom{8}{4}}{\binom{14}{4}}=0{,}930$

b. $\dfrac{\binom{9}{4}+\binom{9}{3}\cdot\binom{5}{1}}{\binom{14}{4}}=0{,}545$

c. $\dfrac{\binom{8}{4}}{\binom{14}{4}}=0{,}070$

d. $P(\text{minder dan 3 zwart})=1-P(\text{3 zwart})=1-\dfrac{\binom{3}{3}\cdot\binom{11}{1}}{\binom{14}{4}}=0{,}989$

**Opgave 9:**

a. $\dfrac{\binom{115}{6}+\binom{115}{5}\cdot\binom{5}{1}}{\binom{120}{6}}=0{,}980$

b. 1 keer 100 of 4 keer 25

$$\frac{\binom{1}{1}\cdot\binom{115}{5}+\binom{4}{4}\cdot\binom{115}{2}}{\binom{120}{6}} = 0{,}042$$

c. minimaal 30 euro aan prijzengeld winnen

$$P(\text{geen verlies}) = 1 - P(0 \text{ of } 25 \text{ euro}) = 1 - \frac{\binom{115}{6}+\binom{115}{5}\cdot\binom{4}{1}}{\binom{120}{6}} = 0{,}062$$

**Opgave 10:**

a. $\left(\frac{2}{6}\right)^4 = \frac{16}{1296} = \frac{1}{81}$

b. $\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$

c. $P(\text{minstens 1 keer meer dan 2 ogen}) = 1 - P(\text{met iedere dobbelsteen 1 of 2})$

$$= 1 - \left(\frac{2}{6}\right)^4 = 1 - \frac{16}{1296} = \frac{1280}{1296} = \frac{80}{81}$$

**Opgave 11:**

a. $\frac{2}{5}\cdot\frac{1}{4}\cdot\frac{1}{3} = \frac{2}{60}$

b. $\frac{2}{5}\cdot\frac{3}{4}\cdot\frac{2}{3} + \frac{3}{5}\cdot\frac{1}{4}\cdot\frac{2}{3} + \frac{3}{5}\cdot\frac{3}{4}\cdot\frac{1}{3} = \frac{27}{60}$

c. $\frac{1}{5}\cdot\frac{2}{4}\cdot\frac{2}{3} + \frac{1}{5}\cdot\frac{2}{4}\cdot\frac{1}{3} + \frac{4}{5}\cdot\frac{2}{4}\cdot\frac{1}{3} + \frac{1}{5}\cdot\frac{2}{4}\cdot\frac{1}{3} = \frac{16}{60}$

d. $\frac{2}{5}\cdot\frac{1}{4}\cdot\frac{1}{3} + \frac{2}{5}\cdot\frac{1}{4}\cdot\frac{1}{3} + \frac{1}{5}\cdot\frac{2}{4}\cdot\frac{1}{3} = \frac{6}{60}$

e. $\frac{3}{5}\cdot\frac{3}{4}\cdot\frac{2}{3} + \frac{2}{5}\cdot\frac{3}{4}\cdot\frac{2}{3} + \frac{3}{5}\cdot\frac{1}{4}\cdot\frac{2}{3} + \frac{3}{5}\cdot\frac{3}{4}\cdot\frac{1}{3} = \frac{45}{60}$

**Opgave 12:**

a. $0{,}83 \cdot 0{,}66 \cdot 0{,}41 = 0{,}225$

b. $0{,}83 \cdot 0{,}66 \cdot 0{,}59 = 0{,}323$

c. $0{,}83 \cdot 0{,}66 \cdot 0{,}41 \cdot 0{,}12 = 0{,}027$

d. $P(\text{minder dan 4 maanden oud}) = 1 - P(\text{minstens 4 maanden}) = 1 - 0{,}027 = 0{,}973$

**Gemengde opgaven hoofdstuk 2.**

**Opgave 14:**

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | | | | | | |
| 5 | | | | | | |
| 4 | x | | | | | |
| 3 | x | x | | | | |
| 2 | x | x | x | | | |
| 1 | x | x | x | x | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave a

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | | | | x | | |
| 5 | | | x | | | |
| 4 | | x | | | | x |
| 3 | x | | | | x | |
| 2 | | | | x | | |
| 1 | | | x | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave b

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | | x | | x | | x |
| 5 | | | | x | | |
| 4 | x | X | x | x | x | x |
| 3 | | | | x | | |
| 2 | | x | | x | | x |
| 1 | | | | x | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |

opgave c

a. $\frac{10}{36}$

b. $\frac{8}{36}$

c. $\frac{15}{36}$

**Opgave 15:**

a. $\frac{71+43}{260} = 0{,}438$

b. $\frac{16+12}{52} = 0{,}538$

c. $\frac{16+12}{71+43} = 0{,}246$

d. $\frac{7}{260} = 0{,}027$

**Opgave 16:**

a. $\frac{9}{30} \cdot 100\% = 30\%$

b. $\frac{12}{100} \cdot 100\% = 12\%$

c. $P(am \mid B) = \frac{12}{30} = 0{,}4$

$P(am) = \frac{45}{100} = 0{,}45$

De kansen zijn niet gelijk, dus zijn de gebeurtenissen niet onafhankelijk.

| | Am | s/f | ac | cu | |
|---|---|---|---|---|---|
| A | 24 | 8 | 4 | 4 | 40 |
| B | 12 | 9 | 6 | 3 | 30 |
| C | 9 | 9 | 6 | 6 | 30 |
| | 45 | 26 | 16 | 13 | 100 |

**Opgave 17:**

a. $\frac{230}{1600} = 0{,}144$

b. $\frac{613}{1600} = 0{,}383$

c. $\frac{140}{613} = 0{,}228$

d. $\frac{140}{576} = 0{,}243$

e. $\frac{\binom{12}{5} \cdot \binom{18}{5}}{\binom{30}{10}}$

f. $\frac{\binom{8}{3} \cdot \binom{22}{7} + \binom{8}{4} \cdot \binom{22}{6}}{\binom{30}{10}} = 0{,}492$

g. $\frac{\binom{13}{4} \cdot \binom{8}{2} \cdot \binom{9}{4}}{\binom{30}{10}} = 0{,}084$

| | lezen | computer | tv | |
|---|---|---|---|---|
| sport | 321 | 115 | 140 | 576 |
| geen sport | 321 | 230 | 473 | 1024 |
| | 642 | 345 | 613 | 1600 |

**Opgave 18:**

a. $\frac{\binom{56}{4}}{\binom{76}{4}} = 0{,}286$

b. $\frac{\binom{20}{2}\cdot\binom{56}{2}}{\binom{76}{4}} = 0{,}228$

c. $\frac{\binom{20}{3}\cdot\binom{56}{1}+\binom{36}{3}\cdot\binom{40}{1}+\binom{12}{3}\cdot\binom{64}{1}+\binom{8}{3}\cdot\binom{68}{1}}{\binom{76}{4}} = 0{,}286$

**Opgave 19:**

a. $\frac{\binom{55}{5}}{\binom{65}{5}} = 0{,}421$

b. $\frac{\binom{6}{1}\cdot\binom{55}{4}}{\binom{65}{5}} = 0{,}248$

c. $\frac{\binom{3}{1}\cdot\binom{6}{1}\cdot\binom{55}{3}}{\binom{65}{5}} = 0{,}057$

d. $\frac{\binom{10}{2}\cdot\binom{55}{3}}{\binom{65}{5}} = 0{,}143$

**Opgave 20:**

a. $\frac{\binom{21}{2}\cdot\binom{99}{10}+\binom{21}{1}\cdot\binom{99}{11}+\binom{99}{12}}{\binom{120}{12}} = 0{,}649$

b. dit is dezelfde vraag als opgave a, dus 0,649

c. $\frac{\binom{32}{3}\cdot\binom{88}{9}}{\binom{120}{12}} = 0{,}269$

**Opgave 21:**

a. kaaa of akaa

$\frac{3}{15}\cdot\frac{2}{15}\cdot\frac{2}{15}\cdot\frac{5}{15}+\frac{5}{15}\cdot\frac{2}{15}\cdot\frac{2}{15}\cdot\frac{5}{15} = \frac{160}{50625} = 0{,}00316$

b. 4 appels of 4 peren of 4 bananen of 4 kiwi's

$\frac{5}{15}\cdot\frac{2}{15}\cdot\frac{2}{15}\cdot\frac{5}{15}+\frac{3}{15}\cdot\frac{4}{15}\cdot\frac{4}{15}\cdot\frac{3}{15}+\frac{1}{15}\cdot\frac{5}{15}\cdot\frac{5}{15}\cdot\frac{1}{15}+\frac{3}{15}\cdot\frac{2}{15}\cdot\frac{4}{15}\cdot\frac{6}{15} = \frac{413}{50625} = 0{,}00816$

c. $\frac{1}{15}\cdot\frac{10}{15}\cdot\frac{10}{15}\cdot\frac{14}{15}+\frac{14}{15}\cdot\frac{5}{15}\cdot\frac{10}{15}\cdot\frac{14}{15}+\frac{14}{15}\cdot\frac{10}{15}\cdot\frac{5}{15}\cdot\frac{14}{15}+\frac{14}{15}\cdot\frac{10}{15}\cdot\frac{10}{15}\cdot\frac{1}{15} = \frac{22400}{50625} = 0{,}44247$

d. $\frac{12}{15}\cdot\frac{13}{15}\cdot\frac{11}{15}\cdot\frac{9}{15} = \frac{15444}{50625} = 0{,}30507$

**Opgave 22:**

a. $\frac{5}{9}\cdot\frac{7}{11}\cdot\frac{9}{13} = \frac{315}{1287} = 0{,}245$

b. rrw of rwr of wrr of rrr

$\frac{5}{9}\cdot\frac{7}{11}\cdot\frac{4}{13}+\frac{5}{9}\cdot\frac{4}{11}\cdot\frac{7}{13}+\frac{4}{9}\cdot\frac{5}{11}\cdot\frac{7}{13}+\frac{5}{9}\cdot\frac{7}{11}\cdot\frac{9}{13} = \frac{735}{1287} = 0{,}571$

**Opgave 23:**

a. $\binom{49}{3} = 18424$

b. $2^{49} = 5{,}6 \cdot 10^{14} > 10^{11}$

c. $1 - P(\text{ geen leesfouten}) = 1 - 0{,}99995^{64} = 0{,}0032$